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Contributions to Modern and Applied Physics

Resolving the Color Confinement Problem

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The unsolved problem of "color confinement" of quarks, implicitly resolved earlier, is explicitly resolved in the Scalar Strong Interaction hadron theory here.

Introduction

The purpose of an earlier and this commentary is to specifically address some unsolved problems in high energy physics [1]. In a previous commentary, the "proton radius puzzle [2], implicitly resolved earlier, was explicitly resolved in the Scalar Strong Interaction Hadron Theory SSI [3]. In this commentary, two more such unsolved problems are considered. The first one is the dark energy problem, which has been explicitly resolved in [4] and [3 Ch 16] in SSI and needs not be repeated here.

The second one is the "color confinement problem", whose resolution has been briefly sketched in [3 Sec. 4.3] and [4 Appendix]. This problem will be commented more extensively here in view of its half century long existence and its place in the history of development of such kind of theories. For these purposes, the above-mentioned sketches are replaced by a first principle´s treatment given in [3 Ch 2] together with additional comments here.

Historical Example

Classical mechanics (CM), i. e., Newtonian mechanics with relativistic extension, has worked well before the atomic era. In atoms, however, the particle energies and angular momenta are much lower. In this region, CM no longer works. The last well-known application of CM to atomic phenomena is the Bohr-Sommerfeld model.

In such low particle energies and angular momenta region, a new theory, quantum mechanics (QM), takes over. While one cannot go from CM to QM which has to be created, the reverse is possible. As the particle energy and angular momenta increase, QM merges into CM.

Here, quantum chromodynamics (QCD) works well at higher energies where it is perturbative, asymptotic freedom [5], [6] holds and quarks are color confined. As the particle energies decreases, asymptotic freedom ceases to hold and QCD becomes nonperturbative and confinement cannot be proved. During the past half century, many attempts have been made to extend the high energy QCD into the nonperturbative low energy region by means of lattice calculations with powerful computers. Although there are reported successes, these, unlike [3 Ch 5], have been unable to account for the basic meson spectra.

Something is basically wrong in such attempts if the simplest meson spectra need huge computers to account for them. Usually, such simple problems are simply explained analytically. Computer calculations enter later in more complicated situations. Newton's equations provide analytical solutions to the planet orbit around the sun, Computers are helpful when corrections due to interactions with other planets are included. In the atomic case, the simple hydrogen atom has been accounted for analytically with the Schrödinger-Dirac equations. As the atoms get heavier and contain more electrons, computers are helpful to deal with the many body problems. These took basically place in 3 decades after the inception of QM from the 1920´s.

Based upon this analogy, QCD at higher energies cannot be pushed too far into the low energy, nonperturbative region. A new theory needs be created for this region, here the Scalar Strong Interaction hadron theory SSI [7, 3]. It has been successful in accounting for the ground state meson spectra and many other hadronic phenomena like the above-mentioned proton radius puzzle and dark matter and energy [3 Ch 16]. Analogous to the merger of QM to CM, SSI leads at higher energies to a QCD Lagrangian [3 Ch 14].

Quark and Meson Equations [3 (2.1.1-2.2.5)] and Confinement in SSI

The starting point is the van der Waerden equations [8], a transformed form of the Dirac equations that is manifestly Lorentz covariant and more suitable for relativistic particles. While Dirac´s wave functions are compatible with the regular representation of the Lorentz group, the van der Waerden spinors are basis vectors generating the fundamental representation of the SL(2,C) Lorentz group.

When applied to the motion of a quark A at x_I interacting with an antiquark B at x_{II} via the potential V_{SB}, these read

$$\partial_I^{a\dot{b}}\chi_{A\dot{b}}(x_I) - iV_{SB}(x_I)\psi_A^a(x_I) = im_A\psi_A^a(x_I) \quad (1a)$$

$$\partial_{I\dot{b}c}\psi_A^c(x_I) - iV_{SB}(x_I)\chi_{A\dot{b}}(x_I) = im_A\chi_{A\dot{b}}(x_I) \quad (1b)$$

$$\Box_I V_{SB}(x_I) = \tfrac{1}{2} g_s^2\left(\psi_B^b(x_I)\chi_{Bb}(x_I) + \psi_B^{\dot b}(x_I)\chi_{B\dot b}(x_I)\right) \quad (2)$$

$$\partial_{II\dot e f}\chi_B^f(x_{II}) - iV_{SA}(x_{II})\psi_{B\dot e}(x_{II}) = im_B\psi_{B\dot e}(x_{II}) \quad (3a)$$

$$\partial_{II}^{\dot d e}\psi_{B\dot e}(x_{II}) - iV_{SA}(x_{II})\chi_B^{\dot d}(x_{II}) = im_B\chi_B^{\dot d}(x_{II}) \quad (3b)$$

$$\Box_{II} V_{SA}(x_{II}) = \tfrac{1}{2} g_s^2\left(\psi_A^b(x_{II})\chi_{Ab}(x_{II}) + \psi_A^{\dot b}(x_{II})\chi_{A\dot b}(x_{II})\right) \quad (4)$$

Conversely, the antiquark B interacts with the quark A via V_{SA}. Here, ∂_I and ∂_{II} refer to differentiations with respect to x_I and x_{II}, respectively. g_s^2 is the scalar strong coupling constant for quark-antiquark interaction. m is the quark mass, $V_{SB}(x_I)$ is the scalar strong interaction potential at x_I generated by quark B at x_{II} and vice versa for $V_{SA}(x_{II})$. Ψ and χ are the left-handed and right-handed, respectively, quark spinors and the dotted and undotted spinor indices $b, e, f\ldots$ run from 1 to 2. The subscripts A and B refer to the quark species.

$$\chi_{A\dot b}(x_I)\chi_B^f(x_I) \to \chi_{\dot b}^f(x_I,x_I), \quad \psi_A^c(x_I)\psi_{B\dot e}(x_I) \to \psi_{\dot e}^c(x_I,x_I) \quad (5)$$

$$\chi_{A\dot b}(x_I)\psi_{B\dot e}(x_{II}) \to \chi_{\dot b\dot e}(x_I,x_{II}), \quad \psi_A^c(x_I)\chi_B^f(x_{II}) \to \psi^{cf}(x_I,x_{II}) \quad (6)$$

$$V_{SA}(x_{II})V_{SB}(x_I) \to \Phi_m(x_I,x_{II}) \quad (7)$$

Here, (5) represents the meson wave functions, (6) the antidiquark and diquark wave functions and (7) the scalar strong interaction potential between A and B. The above-mentioned multiplication gives rise to 3×3=9 equations containing the 2 meson wave functions in (5) and the interquark potential (7) as well a second group containing 2 quark (3), 2 diquark and 2 antidiquark (6) wave functions. Since this second group does not appear in mesons, they are put to 0. Six of the 9 product equations drop out leaving behind the following 3 coupled meson equations

$$\partial_I^{\dot a b}\partial_{I\dot e f}\chi_b^f(x_I,x_I) + \left(m_A m_B - \Phi_m(x_I,x_I)\right)\psi_{\dot e}^a(x_I,x_I) = 0 \quad (8a)$$

$$\partial_{Ibc}\partial_I^{d\dot e}\psi_{\dot e}^c(x_I,x_I) + \left(m_A m_B - \Phi_m(x_I,x_I)\right)\chi_b^d(x_I,x_I) = 0 \quad (8b)$$

$$\Box_I\Box_I\Phi_m(x_I,x_I) = -\frac{g_s^4}{4}\left(\psi^{b\dot a}(x_I,x_I)\chi^*_{\dot a b}(x_I,x_I) + \psi^{*a\dot b}(x_I,x_I)\chi_{\dot b a}(x_I,x_I)\right) \quad (9)$$

Carry out the transformation [3 (3.1.3a)]

$$x^\mu = x_{II}^\mu - x_I^\mu, \qquad X^\mu = (1-a_m)x_I^\mu + a_m x_I^\mu \quad (10)$$

where a_m is a real constant. Conventionally, a_m=1/2 if the quark and antiquark have the same mass. Since x_I and x_{II} are invisible, these masses cannot be measured so that a_m is a free parameter at this stage. The meson laboratory coordinate X is observable but the relative coordinate x is a hidden variable.. "Hidden" variable has been proposed by Einstein, Podolsky and Rosen in 1935 and D. Bohm in 1952 in connection with quantum mechanics, well before the quark era from the 1960´s and the dominating role it plays in SSI [3].

The interquark potential Φ_m depends only upon the distance $r=|\underline{x}|$ between both quarks. The meson wave functions on the right side of (9) contain the dependence $1/\Omega$, the inverse of the volume of the meson in the laboratory space $\underline{X}$. At rest, $\Omega\to\infty$ and the right side of (9) vanishes so that it reduces to [3 (3.2.8a].

$$\Delta\Delta\Phi_m(r)=0, \quad \Delta=(\partial/\partial\underline{x})^2, \quad \Phi_m(r)=d_m/r+d_{m0}+d_{m2}r^2 \quad (11)$$

where the d´s are integration constants. Choosing $d_{m2}=-d_h^2$ [3 (3.2.21)], this term provides a confining potential that leads the wave functions Ψ and χ in (8) $\propto exp(-d_h r^2/2)$ in [3 (4.3.4)] and hence confined.

This potential leads to a simple algebraic expression that approximately accounts for the ground state pseudoscalar and vector meson masses [9 Table A1]. No lattice calculation on computer is needed.

This confinement arises from the 4th order (9), which in its turn depends upon the multiplication of (1), (2) and (3), (4) to go from the invisible quarks to observable mesons via (10). This is a mathematical result in the hidden relative space $\underline{x}$ originating from the scalar quark-antiquark strong interactions V_{SA} and V_{SB} at the quark level in (1)-(4).

If there were no such potentials, the meson equations (8) can be decoupled back to the quark equations (1) and (3). We would then not exist. That there is a material universe requires that quarks interact with each other, here via (2) and (4).

At higher energies, QCD color confinement may enter. But then, the inhomogeneous term on the right of (9) may also eventually enter and affect confinement.

Confinement of Baryon Wave Functions [3 Ch 9-12]

A ground state baryon consists of a quark interacting with a diquark, which replaces the antiquark in the treatment of mesons above. The 4th order (11) is now replaced by the 6th order [3 (10.2.2a)]

$$\Delta\Delta\Delta\Phi_b(r)=0, \quad \Phi_b(r)=d_b/r+d_{b0}+d_{b1}r+d_{b2}r^2+d_{b4}r^4 \quad (12)$$

where the d_b´s are again integration constants. When applied to data, however, it turns out that d_{b4}=0 and the confining potential has the same form as that for mesons in (11), albeit with different value.

Errata:"Weyl" should read "van der Waerden" in [3]

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