F. C. Hoh., (2024). Contrib Mod Appl Phys, 1(1): 104
https://doi.org/10.33790/cmap1100104

Contributions to Modern and Applied Physics

Charged and Neutral Pion Mass Difference

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Article Details
Article Type: Commentary Article
Received date: 12th August, 2024
Accepted date: 21st August, 2024
Published date: 23rd August, 2024
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Citation: Hoh, F. C., (2024). Charged and Neutral Pion Mass Difference. Contrib Mod Appl Phys 1(1):104. doi: https://doi.org/10.33790/cmap1100104.

Abstarct

The mass difference between charged ($\pi^{\pm}$) and neutral (π^0) pions is estimated to be ≈4.3 MeV, close to the measured 4.6 MeV, in the Scalar Strong Interaction Hadron Theory (SSI) using a "marble" model, The measured charge radius r_m≈0.66 fm gives a far too small classical value e^2/r_m≈2.2 MeV. This large mass difference is mostly tied to the strong interaction potential between the quarks in their invisible relative space.

Introduction

The mass difference between the charged and neutral pions is Δm_π=4.5936 MeV [1], Table A1, has not been accounted for in any first principles´ theory. This value far exceeds the classical charge mass of e^2/r_m =2.18 MeV where r_m =0.659 fm is the $\pi^{\pm}$ charge radius [1]. The standard model SM cannot explain this discrepancy.

Ground state meson spectra have been relatively successfully accounted for in SSI [2] as is seen in Table A1 [2 (5.1.1)]. There is however an exception; the predicted $\pi^{\pm}$-π^0 mass difference ≈2.5 MeV, like 2.1 MeV [2 (5.1.2)] used to represent the charge contribution to meson mass, is far less than data 4.6 MeV.

In this paper, this mass difference Δm_π will be estimated from the first principles´ SSI, which has been successful in accounting for many hadron data not accounted for in the SM. In Sec, 2, a "Marble" model for $\pi^{\pm}$ is proposed. The SSI meson equations [2 Ch 2 [3] are generalized to include electromagnetic gauge fields as perturbations. These are then applied to the pions in Sec. 3, where the mass difference between $\pi^{\pm}$ and π^0 is estimated. The results are discussed in Sec. 4. Some necessary underlying material in SSI are provided in the Appendix.

The "Marble" Model

The pion beta decay $\pi^- \to e + \bar{\upsilon}_e$ suggests that $\pi^{\pm}$ and π^0 have the same strong interaction, the mutual interaction of the u and d quarks, irrespective their charges. This strong attraction takes place in the relative, "hidden" space x^μ between the quarks separated from the electromagnetic interactions between quarks in different hadrons in the visible laboratory space X^μ (A2).

SSI contains both x^μ and X^μ intermixed and the general problems are rather complicated. So far, only problems containing x^μ and X^0, the laboratory time in the simple form of $\exp(-iE_0X^0)$, have been treated. Here, E_0 is the π^0 mass and the laboratory space $\underline{X}$ enters.

There is only one data point in this problem, namely r_m=0.659 fm [1] in $\underline{X}$ space. The simplest assumption is to represent $\pi^{\pm}$ as a "marble" with this radius evenly filled with the charge ±e in $\underline{X}$ space. Now the measured r_m=0.659 fm is generally some mean value of many measurements corresponding to different marble sizes. Assume that these charge distributions are normally distributed, they are converted to a "marble" having sharp boundaries with radius R_m= $r_m\sqrt{\pi}/2$=0.584 fm [4 (2.1)]. This leads to a mass difference of 2.47 MeV which is still much less than Δm_π.

In the marble model, the form in $\underline{X}$ mentioned above (A3) is replaced by a step function

$\Psi(|\underline{X}|=R)$ = 1 (unit length)$^{(-3/2)}$ for $R < R_m$ and 0 for $R \geq R_m$ (2.1)

while keeping the time dependence $exp(-iE_0X^0)$. The ansatz (A3) is then replaced by

$$\psi^{a\dot{e}}(\underline{x}) \to \psi^{a\dot{e}}(\underline{x})\,\Psi(R)exp(-iE_0X^0) \to \delta^{a\dot{e}}\psi_0(r)\,\Psi(R)exp(-iE_0X^0)$$
$$\chi_{\dot{b}f}(x) \to \chi_{\dot{b}f}(\underline{x})\,\Psi(R)exp(-iE_0X^0) \to \delta_{\dot{b}f}\chi_o(r)\,\Psi(R)exp(-iE_0X^0)$$
(2.2)

where $r=|\underline{x}|$. The effect of quark charges, at first for π^+, can now be introduced via the conventional minimal substitutions

$$\partial_I^{a\dot{b}} \to \partial_I^{a\dot{b}} + \frac{1}{2}q_u A^{a\dot{b}}(X), \qquad \partial_{II}^{f\dot{e}} \to \partial_{II}^{f\dot{e}} - \frac{1}{2}q_d A^{f\dot{e}}(X)$$
$$\partial_{I\dot{g}a} \to \partial_{I\dot{g}a} + \frac{1}{2}q_u A_{\dot{g}a}(X), \qquad \partial_{II\dot{e}d} \to \partial_{II\dot{e}d}^{f\dot{e}} - \frac{1}{2}q_d A_{\dot{e}d}(X)$$
(2.3)

in (A1). q_u=2e/3 and q_u=–e/3. As there is no magnetic field only the time component $A_0(\underline{X})$ of the vector potential enters. In the marble model, $A_0(\underline{X})\to A_0(R)$ satisfies

$$\left(\frac{\partial}{\partial \underline{X}}\right)^2 A_0(R) = 4\pi\rho\Psi^2(R) = \frac{3(q_u-q_d)}{R_m^3},\quad A_0(R) = \frac{e}{2R_m^3}R^2 \text{ for } R<R_m \text{ and } \frac{e}{2R_m} \text{ for } R\geq R_m \quad (2.4)$$

where ρ is the charge density in the marble and e is the meson charge. With the introduction of $A_0(R)$, the meson energy E_0 in (2.2) becomes E_0+E_1, where E_1 is the effect of the perturbation A_0. With these preliminaries, the differential operators in (A1) can be written:

$$\partial_I^{a\dot{b}} \to D_I^{a\dot{b}} = \left[\frac{i}{2}\delta^{a\dot{b}}(E_0 + E_1 + q_u A_0(R)) + \underline{\sigma}^{a\dot{b}}\left(-\frac{\partial}{2\partial \underline{X}} + \frac{\partial}{\partial \underline{x}}\right)\right]$$
(2.5a)

$$\partial_{II}^{f\dot{e}} \to D_{II}^{f\dot{e}} = \left[\frac{i}{2}\delta^{f\dot{e}}(E_0 + E_1 - q_d A_0(R)) + \underline{\sigma}^{f\dot{e}}\left(-\frac{\partial}{2\partial \underline{X}} - \frac{\partial}{\partial \underline{x}}\right)\right]$$
(2.5b)

$$\delta_{II\dot{e}d} \to D_{II\dot{e}d} = \left[\frac{i}{2}\delta_{\dot{e}d}(E_0 + E_1 - q_d A_0(R)) + \underline{\sigma}_{\dot{e}d}\left(\frac{\partial}{2\partial \underline{X}} - \frac{\partial}{\partial \underline{x}}\right)\right]$$
(2.5c)

$$\delta_{I\dot{g}a} \to D_{I\dot{g}a} = \left[\frac{i}{2}\delta_{\dot{g}a}(E_0 + E_1 + q_u A_0(R) + \underline{\sigma}_{\dot{g}a}\left(\frac{\partial}{2\partial \underline{X}} + \frac{\partial}{\partial \underline{x}}\right)\right]$$
(2.5d)

(A1) with (2.2) now become

$$D_I^{a\dot{b}}\delta_{\dot{b}f}\chi_o(r)D_{II}^{f\dot{e}} - (M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\psi_0(r) = 0$$
(2.6a)

$$D_{I\dot{g}a}\delta^{a\dot{e}}\psi_0(r)D_{II\dot{e}d} - (M_m^2 - \Phi_m(r))\delta_{\dot{e}d}\chi_o(r) = 0$$
(2.6b)

The common factor $\Psi(R)exp(-i\,(E_0+E_1)X^0)$ attached to the ψ_0 and χ_o, as in (2.2), has been dropped. The D operators are arranged such that the spinor indices couple to each other sequentially with the convention that operators on the right side of the wave functions $\chi_o\,(r)$ and $\psi_0\,(r)$ operate backwards towards left on them.

Since the D_I and D_{II} operators now depend also upon the visible laboratory $\underline{X}$, they no longer commute as the ∂_I and ∂_{II} do in (A1). Thus, the order of operation makes difference. For example, performing the D_I operation first followed by D_{II} in (2.6) will lead to different result from that obtained by $D_I \leftrightarrow D_{II}$.

Following the example of calculation of magnetic moment from the Dirac equation, Operate (2.6a) by $D_{I\dot{g}a}$ from the left and $D_{II\dot{e}d}$ from the right, one finds

$$D_{I\dot{g}a}D_I^{a\dot{b}}\delta_{\dot{b}f}\chi_o(r)D_{II}^{f\dot{e}}D_{II\dot{e}d} - D_{I\dot{g}a}(M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\psi_0(r)D_{II\dot{e}d} = 0 \quad (2.7)$$

There are now 4 possible combinations:

$$D_{I\dot{g}a}\left\{\left[\left(D_I^{a\dot{b}}\delta_{\dot{b}f}\chi_0(r)\right)D_{II}^{f\dot{e}}\right]D_{II\dot{e}d}\right\} + D_{I\dot{g}a}\left\{(M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\chi_0(r)D_{II\dot{e}d}\right\} = 0 \quad (2.8)$$

$$D_{I\dot{g}a}\left\{\left[D_I^{a\dot{b}}\left(\delta_{\dot{b}f}\chi_0(r)D_{II}^{f\dot{e}}\right)\right]D_{II\dot{e}d}\right\} + D_{I\dot{g}a}\left\{(M_m^2 - \Phi_m(r))\delta^{a\dot{e}}\chi_0(r)D_{II\dot{e}d}\right\} = 0 \quad (2.9)$$

$$\left\{D_{I\dot{g}a}\left[\left(D_I^{a\dot{b}}\delta_{\dot{b}f}\chi_0(r)\right)D_{II}^{f\dot{e}}\right]\right\}D_{II\dot{e}d} + \left\{D_{I\dot{g}a}\left(M_m^2 - \Phi_m(r)\right)\delta^{a\dot{e}}\chi_0(r)\right\}D_{II\dot{e}d} = 0 \quad (2.9a)$$

$$\left\{D_{I\dot{g}a}\left[D_I^{a\dot{b}}\left(\delta_{\dot{b}f}\chi_0(r)D_{II}^{f\dot{e}}\right)\right]\right\}D_{II\dot{e}d} + \left\{D_{I\dot{g}a}\left(M_m^2 - \Phi_m(r)\right)\delta^{a\dot{e}}\chi_0(r)\right\} \quad D_{II\dot{e}d} \quad = 0$$
(2.9b)

The operations in the parentheses (…) are performed first followed by those in brackets […] and then those in the braces {….}

To zeroth order, (2.8) is equivalent to (A1) which led to the ground state meson spectra predictions in [3 Ch 5].

First Order Terms and Predicted Results

Our task is to obtain the dependence of the first order E_1 on qA_0 from (2.8-9). This dependence takes place in the laboratory space $\underline{X}$ so that the hidden $\underline{x}$ dependence can be removed by carrying out the contraction

$$\int d\underline{x}^3\ \delta^{d\dot{g}}\ \chi_0(r)(2.8, 2.9)/\int d\underline{x}^3\ \chi_0^2(r) \quad (3.1)$$

Using the Dirac formula

$$(\underline{\sigma}^{a\dot{b}}\underline{a})(\underline{\sigma}_{\dot{b}c}\underline{b}) = \delta_c^a(\underline{a}\underline{b}) + i\underline{\sigma}_c^a(\underline{a}\times\underline{b}),$$
(3.2)

the bracketed expressions in (2.8a) and (2.8b) become respectively

$$\left[\left(D_I^{a\dot{b}}\delta_{\dot{b}f}\chi_0(r)\right)D_{II}^{f\dot{e}}\right] =$$

$$\delta^{a\dot{e}}\left[-(M_m^2 - \Phi_{mav}) - E_0\left(\frac{E_1}{2} + \frac{e}{8R_m^3}(q_u - q_d)R^2\right)\right] + \underline{\sigma}^{a\dot{e}}\,\frac{i}{2}\left[-q_u\underline{X} - \frac{e}{2R_m^3}(q_u + q_d)R^2\frac{\partial}{\partial \underline{x}}\right]$$
(3.3a)

$$0$$

$$\left[D_I^{a\dot{b}}\left(\delta_{\dot{b}f}\chi_0(r)D_{II}^{f\dot{e}}\right)\right] =$$

$$\delta^{a\dot{e}}\left[-\left(M_m^2-\Phi_{mav}\right)-E_0\left(\frac{E_1}{2}+\frac{e}{8R_m^3}(q_u-q_d)R^2\right)\right]+\underline{\sigma}^{a\dot{e}}\frac{i}{2}\left[q_d\underline{X}-\frac{e}{2R_m^3}(q_u+q_d)R^2\frac{\partial}{\partial\underline{x}}\right]$$

(3.3b)

$$\Phi_{mav} = \int d\underline{x}^3\, \chi_0^2(r)\Phi_m(r) / \int d\underline{x}^3\, \chi_0^2(r) = 0.5361 \;\; GeV^2 \quad (3.4)$$

The r dependence in (A7) is very weak. These both expressions differ only in the triplet term in which $q_u \rightarrow -q_d$, just like that between (2.5a) and (2.5b).

In this contraction, terms linear in $\partial/\partial\underline{x}$ drop out. Terms containing Δ are removed via (A6) and (A7). The first order terms in (2.8) are collected to singlet terms $\delta_{gd}(\ldots)$; the triplet terms $\underline{\sigma}_{gd}(\ldots)$ drop out due to (3.1). Collecting the the first order E_1 and q terms, it was found, after some algebra, the mass differences

$$\Delta m_\pi = E_1 = -\frac{e}{R_m^3}\left(\frac{q_u+q_d}{8(M_m^2-\Phi_{mav})}+\frac{q_u-q_d}{4}R^2\right) = 4.306 \text{ MeV} \quad \text{for } R^2 \rightarrow (R^2)_{av} = R_m^2/2$$

(3,5a)

$$\Delta m_\pi = E_1 = -\frac{e}{R_m^3}\left(-\frac{q_u+q_d}{8(M_m^2-\Phi_{mav})}+\frac{q_u-q_d}{4}R^2\right) = -4.922 \text{ MeV} \quad \text{for } R^2 \rightarrow (R^2)_{av} = R_m^2/2$$

(3,5b)

for the sequences (2.8a) and (2.8b), respectively. Similarly, (2.9a) and (2.9b) leads to the same results but with (3.5a)↔(3.5b).

The predicted –4.922 MeV is rejected. The predicted 4.3 MeV is close to the measured value of 4,6 MeV, differing by 7%. The result (3.5a) was derived for π^+ but also holds for π^- as it depends only upon the square of the charges. If the R^2 in (3.5) → $-4R_m^2$, it contributes the usual classical e^2/r_m =2.18 MeV mentioned in the Introduction. This term originates from the Laboratory $\underline{X}$ and is small compared to the term $\propto (q_u+q_d)$, which originates in the hidden, relative space $\underline{x}$.

Evaluation of The Result and Implications

The presence of $|\underline{X}|$=R in (3.5) coming from the potential $A_0(R)$ in (2.4) reminds one of that the $\underline{X}$ dependence of (2.6) remains not dealt with. $\partial/\partial\underline{X}$ operating on the step function (2.1) vanishes for $R< R_m$ and $R>R_m$ but diverges for $R=R_m$ on the surface of the marble. This problem has not been investigated due to its difficulty. These show that this marble model is not fully compatible with the basic equations (2.6). Thus, this model can only provide an estimate of the mass difference Δm_π, not an exact prediction. However, this estimate should be rather good because the R^2 term, approximated by its mean value of $R_m^2/2$ in (3.5), makes up only ~6% of Δm_π.

Δm_π is largely controlled by the strong interaction Φ_{mav} in the hidden space. This large Φ_{mav} is also responsible for the low pion masses, as is shown beneath Table A2. Further, Δm_π depends 94% upon $q_u+q_d=e/3$ only 4% upon the conventional q_u-$q_d=e$. This shows that the strong interaction between the quarks also controls Δm_π . Our present electrostatic conception of Δm_π is no longer valid.

As it can be seen above Sec. 4, this 4% parts is 8 times smaller and of opposite sign relative to the classical value e^2/R_m. It may also be noted that if the first – sign in (3.5) is changed to +, then (3.5a) will give –4.306 and (3.5b) +4.922 Mev, respectively. The mean value of 4.922 and 4.306 is 4.6145 Mev, only 0.45% off the measured 4.5936 MeV. This is equivalent to dropping the R^2 terms originating in the $\underline{X}$ space in (3.5). Only the hidden space part physics contributes; the laboratory space contribution is averaged out.

If the anti-d quark in (3.5) is replaced by an anti-s quark, It may formally be applied to the kaon system $K^\pm$-K^0. (3.5) yields a correction of $\approx \pm 10$ MeV. It is bigger than the predicted 6,1 MeV and measured –3.93 MeV in Table A1 below. But these values are of the same magnitude and bigger than those for the pions. A basic difference here is that, while $\pi^\pm$ and π^0 have the same quark content and same strong interaction, as was mentioned in Sec. 2, $K^\pm$ and K^0 have different quark contents, contains a heavier s quark and hence possibly have somewhat different strong interactions. Further, these kaon masses have already been used there to fix the quark masses in Table A2 and helped to determine the predicted values in Table A1. Thus, (3.5) cannot be applied to them without investigation. These remarks also apply to the D and B mesons in Table A1.

Appendix SSI Meson Equations and Results

Some results of SSI are reproduced below. The equations of motion of mesons was first written down in [3]

$$\partial_I^{a\dot{b}}\partial_{II}^{f\dot{e}}\chi_{\dot{b}f}(x_I,x_{II}) - \left(M_m^2 - \Phi_m(x_I,x_{II})\right)\psi^{a\dot{e}}(x_I,x_{II}) = 0$$

(A1a)

$$\partial_{I\dot{b}c}\partial_{II\dot{e}d}\psi^{c\dot{e}}(x_I,x_{II}) - \left(M_m^2 - \Phi_m(x_I,x_{II})\right)\chi_{\dot{b}d}(x_I,x_{II}) = 0$$

(A1b)

$$M_m^2 = \frac{1}{4}(m_1+m_2)^2$$

(A1c)

which is reproduced from [2 (2.3.22), see also 4 (A1)]. Here, x_I is the coordinate of the quark and x_{II} that of the antiquark. χ is a right-handed two spinor and Ψ the left-handed one. The undotted and dotted spinor indices a, b…f run from 1 to 2. m_1 and m_2 are the quark masses. Φ_m is the strong interquark interaction.

SSI respects the observation that quarks are invisible and the transformation

$$x^\mu = x^\mu_{II} - x^\mu_I, \qquad X^\mu = (1 - a_m)x^\mu_I + a_m x^\mu_{II} \quad \text{(A2)}$$

is made. where a_m is a real constant [2 (3.1.3a), 4 (A4)]. Conventionally, a_m=1/2 if u and d have the same mass. Since x_I and x_{II} are invisible, these masses cannot be measured so that am is a free parameter. The meson laboratory coordinate X^μ is observable but the relative coordinate x^μ is a hidden variable. "Hidden" variable has been proposed by Einstein, Podolsky and Rosen in 1935 and D. Bohm in 1952 in connection with quantum mechanics, well before the quark era from the 1960´s and the dominating role it plays in SSI [2].

For free mesons, (A2) will cause χ and Ψ to contain X^μ dependence in the form of $(1/\Omega)exp(i\underline{K}\underline{X}-iE_0X^0)$ where E_0 is the meson mass, $\underline{K}$ its momentum which → 0 in the rest frame. The normalization volume $\Omega \to \infty$ in [2 (3.1.5-7, 9)]. This ansatz removes the $\underline{X}$ dependence. Since the pions are pseudoscalar, they are represented by the singlets χ_0 and $\Psi_0 = -\chi_0$(A6). The vector part of the wave function $\underline{\Psi}$ and χ representing the vector mesons are dropped. The x^μ dependence part read [2 (3.1.9), 4 (A6)]

$$\psi^{c\dot{e}}(\underline{x}) = \delta^{c\dot{e}}\psi_0(\underline{x})\,exp(i\omega_0 x^0)$$
$$\chi_{\dot{b}f}(x) = \delta_{\dot{b}f}\chi_o(\underline{x})\,exp(i\omega_0 x^0), \quad \text{(A3)}$$

where ω_0 is the relative energy between the quarks.

The two unknown parameters a_m and ω_0 must cancel each other via the relation

$$a_m = 1/2 + \omega_0/E_0 \qquad \text{(A4)}$$

given in [3] and [2 (3.1.10a), 4 (A6)]. During the short life of the pions, ω_0=0 and a_m=1/2. For protons in hydrogen atoms in rarified galactic space, a_m<1/2 or >1/2 causes the relative energy ω_0 between the diquark and quark of proton to become dark matter or dark energy, respectively [2 Ch 15-16].

The interquark strong potential Φ_m in (A1) holding the quarks together depends only upon their hidden separation $|\underline{x}|$=r. It satisfies [2 (3.1.14)]

$$\Delta\Delta\Phi_m(r) = 0, \quad \Delta=(\partial / \partial\underline{x})^2 \qquad \text{(A5)}$$

Hence, χ_0 and Ψ_0 are also functions of r. With these preliminaries, $\partial_I = \partial/\partial_{xI}$ and $\partial_{II} = \partial/\partial_{xII}$ in (A1) can be expressed in terms of X^μ and x^μ and (A1) can be reduced to the pseudoscalar meson radial equation [2 (3.2.3b, 3.2.8a, 3.2.21), Table 5.2]

$$(\Delta + E_0^2/4 + \Phi_m(r) - M_m^2)\chi_0(r) = 0, \qquad \psi_0(r) = -\chi_0(r) \qquad \text{(A6)}$$

$$\Phi_m(r) = d_{m1}/r + d_{m0} - d_h^2 r^2, \quad d_{m1}=0, \quad d_{m0} = 0.64113\ \text{GeV}^2, \quad d_h=0.07\ \text{GeV}^2 \qquad \text{(A7)}$$

$$\chi_0(r) = \frac{1}{\sqrt{\Omega}}\alpha_{00}\,exp\left(-\frac{d_h}{2}r^2\right), \alpha_{00} = \left(\frac{d_h}{\pi}\right)^{3/4} = 0.0577\text{GeV}^{3/2} \qquad \text{(A8)}$$

The three d constants in (A7) were originally unknown integration constants in the solution of the 4th order (A5). However, (A6) only allows one r dependent term for discrete, terminated series solutions. The d_{m1} term was chosen first but was later replaced by the d_h term which leads to confinement (A8). d_{m0} and 5 quark masses in Table A2 are to be fixed by 6 pseudoscalar meson masses in Table A1. (A1) then via (A6) yields the mass formula

$$E_J = \pm\sqrt{(m_p + m_r)^2 - 4d_{m0} + 8d_h\left(J + \frac{3}{2}\right)} \qquad \text{(A9)}$$

[2 (5.1.1)] where J=0 and 1 refers to pseudoscalar and vector mesons, respectively. The subscripts denote quark species. (A9) is a purely strong interaction result from the hidden space and is independent of the quark charges. It leads to the (A9) results in Table A1 and are to be compared with the E_{m0}–2.1 and E_{m1} lines there.

	$*\pi\pm$	π^0	$*K^\pm$	$*K^0$	$D^\pm$	$*D^0$	$*D_s^\pm$	$B^\pm$	$*B^0$	
B_s^0 E_{m0}	139.57	134.98	493.68	497.61	1869.7	1864.8	1968.4	5279.3	5279.7	5366.9
E_{m0}–2.1	137.47	134.98	491.58	497.61	1867.6	1864.8	1966.3	5277.2	5279.7	5366.9
(A9)	139.04	137.52	491.86	497.96	1867.3	1864.7	1966.4	5276.9	5279.1	5363.3
	ρ^+	ρ^0	$K^{*\pm}$	K^{*0}	D^{*+}	D^{*0}	$D_s^{*\pm}$	B^*	B^*	
B_s^{*0} E_{m1}	775.1	775.1	891.76	895.6	2010.3	2006.9	2112.2	5324.7	5325	5415
(A9)	761.1	761.1	895.5	898.8	2011.7	2009.2	2104	5329.6	5331.8	5415.2
$2d_h$	0.1453	0.1453	0.1379	0.1386	0.1364	0.1375	0.1467	0.1202	0.1i94	0.1308

Table A1. E_{m0} and E_{m1} are data [1] in MeV. 2.1 is correction due to meson charge.

$2d_h = (E_{m1}^2 - (E_{m0} - 2.1)^2)/4$ and has been taken to be 0.14 GeV^2 as an average. A * mark on the left denotes that this pseudoscalar meson´s mass has been used as input to fix the quark masses shown in Table A2 [2 Table 5.2].

(A9) values are sensitive to the choice of $2d_h$. If the chosen 0.14 is smaller by 0.05%, the (A9) value 139.04 for $\pi^\pm$ will decrease to 137.47 in agreement with data. The large difference between (A9) and data E_{m1} for ρ may be due to its large width. One also sees that the flavor independence of $2d_h$ holds well despite the large mass differences between the π and the B_s mesons. The predicted $\pi^\pm$-π^0 mass difference ≈ 2.5 MeV is much smaller than data 4.6 MeV and is the main subject of this paper.

$m_1=m_u$, $m_2=m_d$, $m_3=m_s$, $m_4=m_c$, $m_5=m_b$					
m_1(GeV)	m_2-m_1	m_3	m_4	m_5	d_{m0}(GeV^2)
0.6592	0.00215	0.7431	1.6215	4.7786	0.64113

Table A2. Quark masses and d_{m0} obtained from data in Table A1

The relatively large d_{m0} value nearly balances off the other two positive terms in (A9) for the π´s and makes them much smaller their vector counterpart, the ρ´s. As the quark masses increase, towards the right in Table A1, this term becomes less important such that the B* and B masses differ insignificantly.

ERRATUM: “Weyl” should read “van der Waerden” in [4]

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