Journal of Comprehensive Pure and Applied Mathematics

Stochastic Control in Determining a Soccer Player's Performance

Paramahansa Pramanik
Department of Mathematics and Statistics, University of South Alabama, Mobile, AL, 36688, United States.

Article Details
Article Type: Commentary Article
Received date: 25th November, 2024
Accepted date: 06th December, 2024
Published date: 09th December, 2024
***Corresponding Author:** Paramahansa Pramanik, Department of Mathematics and Statistics, University of South Alabama, Mobile, AL, 36688, United States.
Citation: Pramanik, P., (2024). Stochastic Control in Determining a Soccer Player's Performance. *J Compr Pure Appl Math, 2*(1): 111. doi: https://doi.org/10.33790/jcpam1100111.

Abstract

This paper explores recent advancements in utilizing the dynamic programming approach to determine optimal stubbornness in professional soccer matches. Traditional discrete methods often become challenging due to the curse of dimensionality. To address this, we employ a stochastic control framework to maximize a soccer player's objective function, with goal dynamics expressed by a stochastic dierential equation. Solving a high-dimensional Hamilton-Jacobi-Bellman (HJB) equation numerically is very complicated. As a solution, a Feynman-type path integral method is proposed as a more ecient alternative.

Key words: Stochastic Dierential Equations; Stochastic Control; Sports Analytics.

Introduction

Soccer is fundamentally a team sport, but evaluating the quality of individual players is just as crucial as assessing team performance. A player's quality is typically represented as a single numeric value, which helps answer questions like identifying the best player or comparing players' abilities. Such inquiries are relevant to fans, players to build their strategies, team management during contract negotiations, and other stakeholders [5]. In soccer, player rankings are traditionally based on the performance observed during a double round robin tournament, where every team competes twice against every other team, once at home and once away. However, conducting a complete round robin tournament is often unrealistic. If not all teams have faced each other, it is possible that the top-ranked team might have beneted from a relatively easier schedule, which could also skew evaluations of its players [19].

Ranking has become a common feature of modern society, evident in everything from league tables for schools and universities to top ten lists of movies, books, and songs. This fascination with rankings is even more pronounced in sports. In fact, the essence of professional sports lies in determining the rankings of competitors. Soccer leagues worldwide operate with this goal, where the team or player ranked rst at the season's end is crowned the champion. This paper explores various methods and challenges associated with developing optimal strategies for players to achieve higher rankings in professional soccer. The two primary objectives for improving player ratings are to acknowledge past performance and predict future success [10].

The key distinction between these two objectives lies in the consideration of randomness when predicting future performance versus rewarding past achievements. When forecasting future performance, it is essential to account for the signicant role that white noise plays in a competitor's results, particularly in goal-scoring. Due to this inherent variability, predictions about future performance should anticipate some degree of regression to the mean, where players who have either outperformed or underperformed are likely to move closer to average performance levels over time [10]. To address this challenge, a stochastic control approach is applied, where each player is assigned an objective function that incorporates a state variable, such as goal dynamics, and control parameters like stubbornness [14, 20]. Players can monitor the complete goal dynamics, which are modeled using a stochastic differential equation (SDE). The primary goal for each player is to optimize their utility while adhering to the constraints imposed by the goal dynamics [13, 22].

Monte Carlo Approach

This method serves as a natural tool for analyzing goal dynamics, as it can handle models with numerous state variables and is particularly eective for calculating path-dependent expectations. In stochastic goal dynamics models, the likelihood of scoring a goal at a given continuous time is governed by a single-variable diusion process. As a result, even when computing expectations based solely on the terminal condition of the process, deterministic approaches that rely on discretizing the partial dierential equation (PDE) can be computationally intensive [4]. For certain path-dependent expectations, deterministic techniques can still be applied by introducing an additional state variable, which, however, increases the complexity.

Recent advancements in Monte Carlo simulation have sparked signicant interest in these techniques. Variance reduction methods, adapted to diusion processes by Newton (1994) [12], enable Monte Carlo approaches to compete eectively with deterministic methods, even for low-dimensional scenarios like the one in this application.

The technique of importance sampling involves adjusting the process (through modications to the drift coecient) to focus on regions that contribute most to the desired expectation. Using Girsanov's transformation, appropriate weighting is applied to account for changes in the drift coecient. The adjustment to the drift is informed by an initial estimate of the expectation [8, 24]. Two primary approximations are typically employed: the large deviation approximation, which is particularly suitable for computing dynamics far from the goal, and a small-noise expansion of the stochastic volatility model around the simpler Gaussian Black-Scholes framework. It is well established that the latter signicantly enhances the accuracy of the Monte Carlo estimator [13, 22].

Stochastic Control

This approach eectively addresses the stochastic dierential equation (SDE) associated with the soccer player's objective function. Utilizing dynamic programming, the methodology derives a variational inequality for the value function, which takes the form of a second-order nonlinear partial dierential equation (PDE) of elliptic, parabolic, or ergodic type [1, 2]. The solutions to these equations yield the optimal level of stubbornness in feedback form, meaning the optimal stubbornness depends on the system's current goal dynamics [3]. However, the value function is often not suciently smooth to satisfy the dynamic programming equation in a strong sense [9, 21]. Consequently, a weak solution formulation becomes necessary, with the concept of viscosity providing a robust framework well-suited for these equations. Furthermore, except in special cases like the Merton problem without transaction costs, explicit solutions to the dynamic programming equations are typically unattainable, necessitating the use of a Feynman-type path integral approach [23].

Using a Feynman-type path integral approach, we begin by formulating a stochastic Lagrangian based on the soccer player's objective function subject to the goal dynamics. Due to the continuous yet non-dierentiable nature of Brownian motion, an It^o process is employed, and a smooth function is derived through the integration factor method. The nite continuous time interval is then divided into numerous small, equal-length intervals. Within each interval, a Feynman action function is constructed. By applying Taylor series expansions and Gaussian integrals, a Wick-rotated Schr-odinger-type equation is obtained. Finally, the optimal level of stubbornness for the soccer player is determined by solving the rst-order derivative with respect to the control variable [15, 16]. Unlike the classical Hamilton-Jacobi-Bellman (HJB) framework, this path integral method avoids the complexity of computing intricate value functions. Additionally, in higher-dimensional goal dynamics, the HJB approach becomes computationally challenging and time-intensive, whereas the path integral method oers a more practical alternative [17, 18].

Conclusion

This paper presents our perspective on constructing an optimal level of stubbornness for a soccer player to enhance their ratings. A central question is whether the system should incorporate past performance or focus on predicting future outcomes. To estimate future performance, we advocate for the use of SDEs. These models eectively adjust estimated abilities toward the average, with the degree of adjustment depending on the evidence of a player's underperformance or overperformance. The development of models to determine optimal stubbornness in soccer is still at an early stage. Several challenges remain unresolved, with one of the most signicant being the integration of o-the-ball movements into the diusion coecient of the SDE. Another complex issue is accurately valuing goalkeepers' contributions [10]. Goalkeepers often operate almost as though playing a dierent sport, with each save essentially preventing a goal. By symmetry, one might argue that a save should carry the same weight as scoring a goal. However, goalkeepers typically make more saves in a match than the total number of goals scored, creating a nuanced challenge in balancing these contributions.

An area that has received relatively little research attention is the analysis of players based on their defensive capabilities. This presents a challenge, as defensive actions lack a straightforward point-based metric, posing a dilemma similar to determining how goalkeepers should be rewarded for making saves [10]. Another promising direction for future research could involve exploring McKean-Vlasov [11] goal dynamics within the framework of a mean eld approach [6, 7]. Given the extensive range of strategies available to players during a match, their interactions could potentially align with mean eld game theory. Since the probability of scoring a goal at any given point in continuous time can be empirically estimated from the marginal distribution of the state variable, this method could serve as a powerful tool for numerical estimation.

References

1. R. Carmona and F. Delarue. (2015). Forward backward stochastic dierential equations and controlled mckean vlasov dynamics. *The Annals of Probability*, pages 2647-2700.
2. R. Carmona, F. Delarue, and A. Lachapelle., (2013). Control of mckean vlasov dynamics versus mean eld games. *Mathematics and Financial Economics, 7*:131-166.
3. M. G. Crandall, H. Ishii, and P.-L. Lions., (1992). User's guide to viscosity solutions of second order partial dierential equations. *Bulletin of the American mathematical society, 27*(1):1-67.
4. E. Fournie, J. Lasry, and N. Touzi., (1997). Monte carlo methods for stochastic volatility models. *Numer-ical methods in nance*, pages 146-164.
5. R. H. Koning., (2017). Rating of team abilities in soccer. In Handbook of statistical methods and analyses in sports, pages 371-388. Chapman and Hall/CRC
6. J.-M. Lasry and P.-L. Lions., (2006). Jeux a champ moyen. i{le cas stationnaire. *Comptes Rendus Mathematique, 343*(9):619-625.
7. J.-M. Lasry and P.-L. Lions., (2006). Jeux a champ moyen. ii horizon ni et contr^ole optimal. Comptes Rendus. *Mathematique, 343*(10):679-684.
8. J.-M. Lasry and P.-L. Lions., (2007). Mean eld games. *Japanese journal of mathematics, 2*(1):229-260.
9. P.-L. Linos., (1983). Optimal control of diustion processes and hamilton-jacobi-bellman equations part i: the dynamic programming principle and application. *Communications in partial dierential equations, 8*(10):1101-1174.
10. I. G. McHale and S. D. Relton., (2017). Player ratings in soccer. In Handbook of Statistical Methods and Analyses in Sports, pages 389-400. Chapman and Hall/CRC.
11. H. P. McKean Jr. (1966). A class of markov processes associated with nonlinear parabolic equations. *Proceedings of the National Academy of Sciences, 56*(6):1907-1911.
12. N. J. Newton. (1994). Variance reduction for simulated diusions. *SIAM journal on applied mathematics, 54*(6):1780-1805.
13. P. Pramanik., (2020). Optimization of market stochastic dynamics. In SN Operations Research Forum, volume 1, page 31. Springer.
14. P. Pramanik., (2021). Optimization of dynamic objective functions using path integrals. PhD thesis, Northern Illinois University
15. P. Pramanik., (2022). On lock-down control of a pandemic model. arXiv preprint arXiv:2206.04248.
16. P. Pramanik., (2022). Stochastic control of a sir model with non-linear incidence rate through euclidean path integral. arXiv preprint arXiv:2209.13733.
17. P. Pramanik., (2023). Path integral control in infectious disease modeling. arXiv preprint arXiv:2311.02113.

18. P. Pramanik., (2023). Path integral control of a stochastic multi-risk sir pandemic model. *Theory in Biosciences, 142*(2):107-142,
19. P. Pramanik, E. L. Boone, and R. A. Ghanam., (2024). Parametric estimation in fractional stochastic dierential equation. *Stats, 7*(3):745.
20. P. Pramanik and A. M. Polansky., (2023). Scoring a goal optimally in a soccer game under liouville-like quantum gravity action. In Operations Research Forum, volume 4, page 66. Springer.
21. P. Pramanik and A. M. Polansky., (2024). Motivation to run in one-day cricket. *Mathematics, 12*(17):2739.
22. P. Pramanik and A. M. Polansky., (2024). Optimization of a dynamic prot function using euclidean path integral. *SN Business & Economics, 4*(1):8.
23. P. Pramanik and A. M. Polansky., (2024). Semicooperation under curved strategy spacetime. *The Journal of Mathematical Sociology, 48*(2):172-206.
24. D. W. Yeung and L. A. Petrosyan., (2006). Cooperative stochastic dierential games, volume 42. Springer